Math 155, *Lecture Notes- Bonds* **Name____________**

Section 8.4 *Trigonometric Substitutions*

When working with integrands that contain the following three expressions,

$$\sqrt{a^2 - u^2},\ \sqrt{a^2 + u^2},\ \text{and}\ \sqrt{u^2 - a^2},$$

(difference of squares or sum of squares),

you should consider applying a trigonometric substitution technique. That is, re-write the integral using trigonometric functions based on a particular right triangle defined by the sides of u and a.

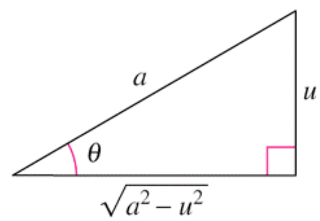
A critical component of this technique is to apply differentiation to our u-substitution function and solve for dx. You might notice that our examples, x is actually a function of θ, so we'll be solving for dx in terms of $d\theta$.

Trigonometric Substitution ($a > 0$)

1. For integrals involving $\sqrt{a^2 - u^2}$, let

$$u = a \sin \theta.$$

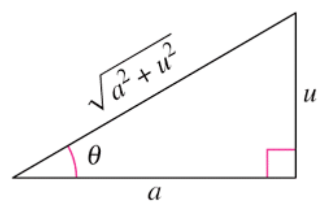
Then $\sqrt{a^2 - u^2} = a \cos \theta$, where $-\pi/2 \le \theta \le \pi/2$.



2. For integrals involving $\sqrt{a^2 + u^2}$, let

$$u = a \tan \theta.$$

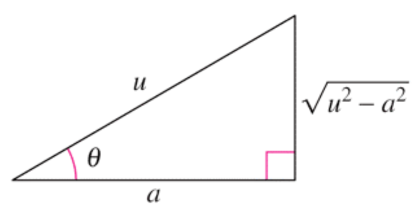
Then $\sqrt{a^2 + u^2} = a \sec \theta$, where $-\pi/2 < \theta < \pi/2$.



3. For integrals involving $\sqrt{u^2 - a^2}$, let

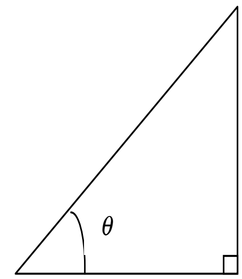
$$u = a \sec \theta.$$

Then $\sqrt{u^2 - a^2} = \pm a \tan \theta$, where $0 \le \theta < \pi/2$ or $\pi/2 < \theta \le \pi$. Use the positive value if $u > a$ and the negative value if $u < -a$.

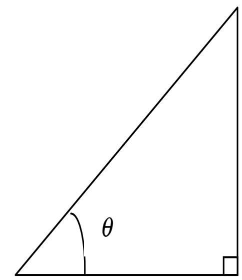


NOTE: We will need to carefully create our right triangle in order to see all of the relevant trigonometric functions. Don't forget to label the side of your triangle using *opp*, *adj*, and *hyp*.

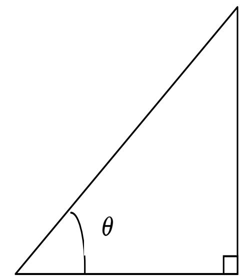
Ex.1 Integrate: $\int \frac{x}{\sqrt{9-x^2}}dx$



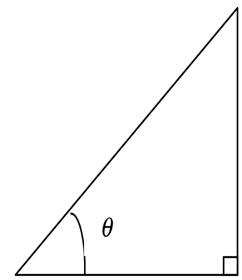
Ex.2 Find the antiderivative: $\int \frac{\sqrt{4x^2+9}}{x^4}dx$



Ex.3 Evaluate: $\displaystyle\int_3^6 \frac{\sqrt{x^2-9}}{x^2}dx$



Ex.4 Integrate: $\displaystyle\int \frac{\sqrt{x^2-4}}{x}dx$



Ex.5 Find the antiderivative: $\int \sqrt{16-4x^2}\,dx$